****King Fahd University of Petroleum & Minerals

College of Computer Science and Engineering

Information and Computer Science Department

Second Semester 162 (2016/2017)

ICS 254 – Discrete Structures 2

Final Exam

Friday, May 26h, 2017

Time: 120 minutes

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ID# |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Section 01 |  | Question # | Max Points | Points Earned |
| Wasfi Al-Khatib |  | 1 [MCQs] | 20 |  |
|  |  | 2 [Graphs] | 30 |  |
| Section 02 |  | 3 [Trees] | 10 |  |
| Faisal Alvi |  | 4 [Automata] | 20 |  |
|  |  | 5 [Automata] | 20 |  |
|  |  | Total | 100 |  |

**Instructions**

1. **Write your name and ID in the respective boxes above and circle your section.**
2. **This exam consists of 9 pages, including this page, containing 5 questions.**
3. **You have to answer all 5 questions.**
4. **The exam is closed book and closed notes. No calculators or any helping aids are allowed.**
5. **Make sure you turn off your mobile phone and keep it in your pocket if you have one.**
6. **The questions are not equally weighed.**
7. **The maximum number of points for this exam is 100.**
8. **You have exactly 120 minutes to finish the exam.**
9. **Make sure your answers are readable.**
10. **If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order not to miss grading it.**

**Q.1: [MCQs: 20 points = 2\*10]: For the following questions write down the correct answer in the table provided on Page 3.**

1. The value of is equal to
	1. 0
	2. 6
	3. 13
	4. 1
	5. none of the above.
2. The decryption function of the affine cypher
	1. is equal to
	2. is equal to
	3. is equal to
	4. is equal to
	5. does not exist because .
3. The value of is:
	1. 0
	2. 1
	3. 10
	4. 12
	5. 13
4. Let . Then, which of the following statements is correct?
	1.
	2. All statements i), ii) and iii) are correct
	3. Only statements ii) and iii) are correct.
5. The relation over the set is
	1. an equivalence relation but not a partial ordering.
	2. a partial ordering but not an equivalence relation.
	3. an equivalence relation, a partial ordering but not a total ordering.
	4. an equivalence relation, a partial ordering and a total ordering.
	5. neither an equivalence relation nor a partial ordering.
6. An undirected simple graph has 7 vertices with degrees 1, 2, 2, 3, 3, 4, 5. The number of edges in the graph is:
	* 1. 7
		2. 8
		3. 9
		4. 10
		5. None of the above
7. Which of the following graphs is bipartite?
	1. *K*3 (complete graph on 4 vertices)
	2. *W*3 (wheel graph on 4 vertices)
	3. *C*3 (cycle graph on 4 vertices)
	4. *Q*3 (n-cube on 4 vertices)
	5. None of the above

Questions 8-9 refer to the following graph:

|  |  |
| --- | --- |
|  | 1. For the graph shown here, all of the following path/circuits exist in the graph EXCEPT:
	* 1. Euler circuit
		2. Euler path
		3. Hamilton Circuit
		4. Hamilton Path
		5. None of the above
 |

1. The chromatic number of the shown graph (above) is:
	* 1. 1
		2. 2
		3. 3
		4. 4
		5. None of the above
2. A binary tree of height 3 has 7 internal vertices and 8 leaves. What is the total number of vertices?
	* 1. 3
		2. 7
		3. 8
		4. 15
		5. None of the above

Write down your answers here:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

**Q.2: [Graphs] [8+7+7+8 = 30 points]**

(a) A simple graph having *e* edges and *v* vertices has the minimum degree *m* of its vertices and the maximum degree *M.* Show that

*m v* ≤ 2 *e*  ≤ *M v*

(b) Are the following two graphs isomorphic? If yes, give a mapping. If no, provide a reason.



(c) Is the following graph planar? If yes, redraw it without any edge crossings. If no, provide a reason.



|  |  |
| --- | --- |
| (d) For the following graph, which of the following paths/circuits exist in the graph. If the path or circuit exists, write down its vertex sequence. If it does not exist, provide a reason. |  |

|  |  |  |
| --- | --- | --- |
|  | Exists (Yes/No) | Path/Circuit (if yes), reason (if no) |
| Euler Circuit |  |  |
| Euler Path |  |  |
| Hamilton Circuit |  |  |
| Hamilton Path |  |  |

**Q. 3 [Trees] [10 points]**

When a particular malware (computer virus) infects a computer on a network, it makes three copies of itself and spreads to exactly three other computers on this network. However, if the virus reaches a fully protected computer, then it does not infect it and does not spread from this protected computer to other computers. If in a network, a virus infects 10,000 computers in all, then how many computers on this network were not infected (assuming that the virus reaches every computer on this network)? Consider this network as a full m-ary tree to derive your answer.

**Q. 4 [Automata] [10+10 = 20 points]**

(a) Determine whether belongs to each of these languages generated by the regular expressions

 i.

 ii.

 iii.

 iv.

 v.

(b) Let . Give a regular expression that describes each of the following regular languages

 i. The set of strings consisting of two ’s, followed by zero or more ’s and ending with an .

 ii. The set of strings ending in and not containing the substring .

(c) Let . Draw a deterministic finite automaton that recognizes the language containing an

 even number of ’s.

**Q. 5 [Automata] [9 + 5 + 6 = 20 points]**

(a) Consider the following deterministic finite automaton (DFA).

i. Formally describe the above DFA.

ii. Describe all strings of length at most 3 that are accepted by the DFA.

iii. Determine the value of

(b) Consider the following nondeterministic finite automaton (NFA).

Showing the propagation of states, determine whether the NFA accepts the string or not.

(c) Draw a non-deterministic finite automaton for the regular expression